Lecture 1

Polar Coordinates

Coming from Cartesian coordinates (x,y), we obtain polar coordinates (r,θ) via

$$r^2 = x^2 + y^2 \quad \text{and} \quad \tan\theta = \frac{y}{x}.$$

and in the reverse direction

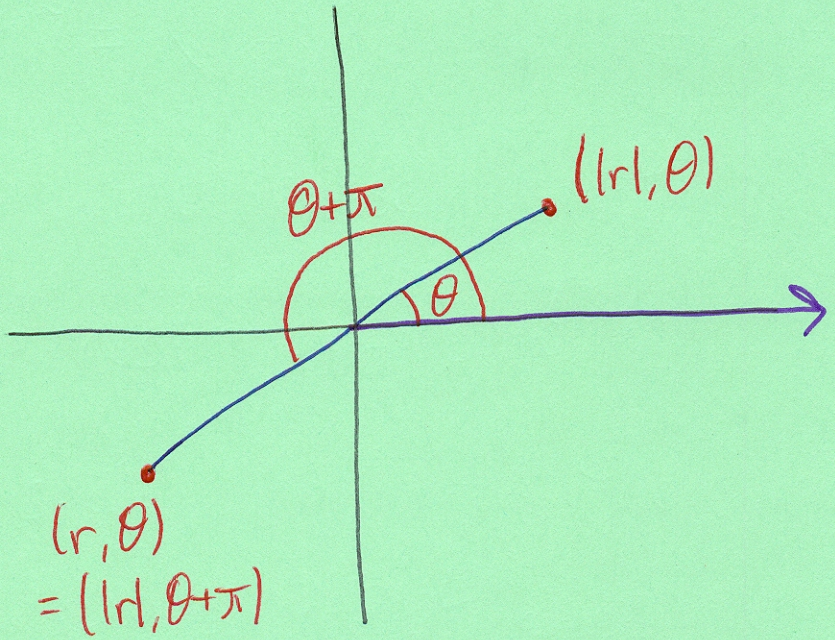
$$x = r\cos\theta, \quad y = r\sin\theta.$$

One should think of the plane in polar coordinates as consisting of an origin $(r=0)$ and a "polar axis" $(\theta=0)$. For the standard xy-plane, these match up as the point $(0,0)$ and the positive x-axis, respectively. Geometrically, r represents the distance from the origin and θ represents the angle between the line connecting the point to the origin, and the polar axis.

We can cover the whole plane by only using $r \geq 0$ and $0 \leq \theta < 2\pi$, but we can make sense of other values of r & θ as well.

If $r<0$, then we have

$$(r,\theta) = (|r|, \theta+\pi)$$



This makes sense with the equations since
$\sin(\theta+\pi) = -\sin\theta$ & $\cos(\theta+\pi) = -\cos\theta$

Graphing Polar Equations

Sometimes, to graph polar equations, it is useful to turn them into Cartesian equations first.

Ex: Graph $r = \sin\theta$

Sol: First, multiply this equation by r:

$$r^2 = r\sin\theta$$

Substitute known quantities:

$$x^2+y^2 = r^2 = r\sin\theta = y$$

$$\Rightarrow x^2+y^2 = y$$

$$\Rightarrow x^2+y^2-y = 0$$

Complete the square:

$$x^2+\left(y-\tfrac{1}{2}\right)^2 = \frac{1}{4}$$

So, this is a circle of radius $\frac{1}{2}$ with center $(0,\frac{1}{2})$.

To verify this, notice that as θ goes from 0 to $\pi/2$, r goes from 0 to 1, so this traces the right edge of the circle. Since r goes from 1 back to 0 as θ goes from $\pi/2$ to π, we get the left edge too.